A Simplified Mixing Length Model of Flame Stability in Swirling Combustion

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A model for the stability swirling flames is presented. The Zukowski-Marble criteria for flame stability behind bluff bodies is extended to flows exhibiting a recirculation region such as observed in strongly swirling jets. Both the recirculation distance and the effective flame speed are shown to be functions of the swirl parameter, S. An effective turbulent diffusivity relationship is obtained from an extension of the Prandtl mixing length theory and shown to be $\varepsilon_m = KR(1+\beta^2 S^2)^{1/2}\bar{U}_\infty$. By characterizing the combustion zone as a free shear layer, an effective flame speed is obtained in terms of the diffusivity expression. The stability of the flame is obtained from the blowoff velocity model $U_{BG} = K'R\cdot\Phi(S)\cdot\bar{U}_\infty{}^2/\alpha$ which is compared with available data. It is concluded that for strongly swirling flows, the principal fluid dynamic variable, S, controls the process. The limits of applicability are reached when the recirculation zone produced by swirl extends to form a rotating Taylor column.

Nomenclature

A', B, C = constants
c_p = specific heat
$\tilde{D}$ = deformation tensor
H = enthalpy
k = thermal conductivity
K = universal constant of mixing
K' = constant in Eq. (20)
l = mixing length
L = recirculation length
$\dot{m}_\infty$ = axial mass flow rate
$\dot{m}_T$ = tangential mass flow rate
N_{Pe} = Peclet number
N_{Pr} = Prandtl number
N_{Re} = Reynolds number based on nozzle diameter
N_{Ro} = V_T/U_∞, Rossby number based on duct diameter
N_{St} = Stability number for flame holding
P = static pressure
ΔP = pressure differential
R = duct radius
r_u = reaction rate
S = Swirl number = V_T/U_∞
s = distance in Eq. (17)
T = temperature
$\bar{U}_\infty$ = axial mean velocity
u' = fluctuating velocity component
$(\overline{u^2})^{1/2}$ = rms velocity
V = volume
V_T = tangential velocity at wall
$\bar{V}_n$ = mean normal propagation velocity in superlayer
v_o = mean flame speed
v_l = laminar flame speed
ω = angular velocity
y = distance normal from wall
α = thermal diffusivity = $k/\rho c_p$
$\varepsilon_m, \varepsilon_D$ = eddy diffusivity (momentum), (mass)
ρ = density
θ, r, z = cylindrical coordinate directions
ν = kinematic viscosity
Ω = rotational velocity
τ = shear stress
Φ = functional relationship in Eq. (20)
ϕ = fuel air ratio

Subscripts
BO = blowoff conditions
∞ = main stream conditions
c = combustion temperature
o = reference state
T = tangential conditions
i, j, k = indices
l = laminar conditions
b = burned conditions
u = unburned conditions
r = radial direction
z = axial direction
θ = angular direction

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Introduction

IN several early studies in which swirling flow was used to promote high intensity combustion, it was found that the stability of the flame could be correlated with the swirl intensity or vortex strength.[1] Recent studies confirm these observations; however, there has been no adequate theory advanced that might permit one to develop a prediction of the effect of swirl on flame stability.[2]

The influence of the swirl on the flame zone is at present qualitative although there is now additional evidence that the flame zone is supported in a region of reverse flow formed along the axis of the burner.[3] Optical examination of this flow region indicates that at a critical ratio of the swirl velocity to the axial flow velocity, an axisymmetric region of reverse flow develops, expands with increasing swirl velocity, and finally disappears at an upper critical value of the swirl.[4]

Empirical data for stability shows that the blowoff velocity is related to the 0.73 power of the swirl or vortex strength and is only slightly dependent on fuel concentration.[5] The proximity of walls does not appear to alter the stability. One may conclude that the phenomena is driven by the vortex alone.

The presence of the reverse flow region is in all respects identical to the reverse flow region observed in cold swirling flows. These flow phenomena have been extensively discussed by Benjamin,[6] Harvey,[7] and others.[8,9] In the presentation that follows, a tentative explanation is provided for the observed enhancement of flame stability in a swirling jet and a semi-empirical model is developed to predict blowoff limits in such augmented flames.

Theoretical Development

One-dimensional theories[10,11] have been successfully applied to predict stability in flames supported on simple shapes. In general, it may be shown that the blowoff limits may be determined from the residence time a mass of reactant spends in the neighborhood of the recirculation wake behind the bluff body. Bespalov[12] has studied a wide range of variables and their effect on stability limits of bluff bodies. They can be summarized as the effect of geometry of the stabilizer, the pressure in the flow, and the scale of turbulence generated in the process. Williams[13] suggests that a dimensionless parameter

$$N_{St} = U_{BO}\alpha\rho/L\rho_u v_o^2 \tag{1}$$

should correlate stability on a flame holder independent of the approach Reynolds number.

This relation is obtained from a one-dimensional analysis of the flame front bounding the recirculation zone. The recirculation region is considered according to Zukowski and Marble[10] to be a well stirred reactor. Application of the equation of continuity across the flame sheet and an energy balance and equating the convected heat to the exothermic heat release within the flame sheet an expression is obtained for the extinction conditions. Thus for a recirculation region of volume, V

$$Vr_u \Delta H = \dot{m}_A C_p(T_b - T_u) \tag{2}$$

The rate of depletion, r_u, is proportional to the concentration of unburned fuel specie and to the combustion temperature; and the mass flow rate, $\dot{m}_A = U_{BO}\cdot\rho A$. It follows that Eq. (2) can be related to a function of the fuel-air ratio, ϕ, the nature of the fuel, and its combustion properties

$$U_{BO}\rho A/V\cdot r_u = \text{const}(T_c, H, \phi) \tag{3}$$

The reaction rate is approximately equal to the square of the mean flame speed

$$r_u \propto v_o^2$$

which yields on substitution with $L = v/A$, the Williams relation. For such flows the characteristic dimension of the holder, R, is directly proportional to the length of the recirculation zone, L. Experimental agreement is obtained by expression $\rho \sim p$, the pressure of the system

$$N_{St} \propto U_{BO}/Rpv_o^2 \tag{4}$$

where the physical properties are taken to depend only on the initial conditions of the unburned reactants.

Although such an approach seems valid for bluff body holder, there is some question whether it is equally applicable to a broader variety of flame holding techniques. Of interest would be its usefulness in explaining the enhanced stability of combustion flows at high swirl values. It is adequate in predicting stability limits, provided that the product Lv_o^2 is reinterpreted in terms of the turbulent fluid dynamics occurring in the free shear layer of the swirling jet.

A strongly swirling jet issuing into an undisturbed region exhibits the characteristics of flow as shown in Fig. 1. For our analysis, we may consider the jet divided into two regimes; an outer free shear region where microeddies contribute to mixing; and an inner one, in which macroscale eddies are important. This inner region provides for the entrainment of fresh reactants and its boundary exhibits a strong velocity gradient necessary for flame retention.

The turbulence in the outer shear layer is highly anisotropic. However, as a result of vorticity generated by tangential velocity components the flow characteristics are similar to a general class of free shear layer problems which qualify for analysis using modified boundary-layer equations. Chigier and Lilley[14] have attempted to solve the complete constitutive set of Reynolds stress equations for the entire flowfield of the jet, by assuming that for small swirl values several approximations are possible. For strong swirl, their analysis does not apply. In the analysis that follows, the outer mixing zone is isolated and studied as a two-dimensionalized region. Although the eddy diffusivity for momentum, heat, and mass are not necessarily equal, some reduction in complexity of the task is possible if they are so chosen. The constitutive Reynolds equations are simplified for this outer region by assuming the flow to be axisymmetric and incompressible. The two shear stresses $\tau_{r\theta}$ and τ_{rz} are shown to be important from order of magnitude considerations. If the tensorial form of the Prandtl mixing length theory is utilized, the momentum shear stress is defined as follows

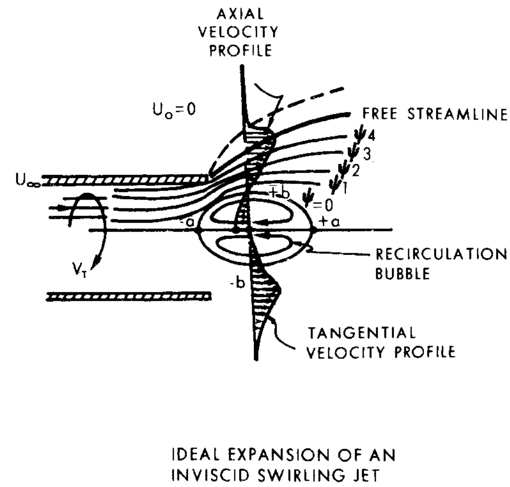

Fig. 1 Schematic representation of a swirling jet.

$$\tau_{ij} = -\rho\overline{u_i u_j} = \rho(\bar{\varepsilon}_m)_{ik}\cdot\bar{D}_{kj} \tag{5}$$

where $\bar{\varepsilon}_m$ is a second-order tensor and $\bar{D}_{kj}$ is the deformation tensor. Assuming axisymmetry, the $\partial/\partial\theta$ and $\partial/\partial z$ components are neglected and assuming $\partial U_r/\partial r \ll \partial U_z/\partial r$, the Prandtl diffusivity, $\bar{\varepsilon}_m$, can be obtained for each stress component which includes the effects of directional flow contributions

$$\varepsilon_{r\theta} = \left\{ l_{r\theta}^2 \left(\left[\frac{\partial \bar{U}_z}{\partial r}\right]^2 + \left[\frac{1}{r}\frac{\bar{U}_\theta/r}{\partial r}\right]^2 \right) \right\}^{1/2} \tag{6}$$

$$\varepsilon_{rz} = \left\{ l_{rz}^2 \left(\left[\frac{\partial \bar{U}_z}{\partial r}\right]^2 + \left[\frac{1}{r}\frac{\partial \bar{U}_\theta/r}{\partial r}\right]^2 \right) \right\}^{1/2} \tag{7}$$

where $l_{r\theta}$ and l_{rz} are the mixing length parameters.

With respect to Eqs. (6) and (7), Lavan[15] has neglected the gradient of the tangential velocity for the case of weakly swirling flow superimposed on an axial flow, while Ragsdale[16] has ignored the axial gradient contribution in his study of a confined vortex. The mixing length parameters, presented in Eqs. (6) and (7) are now utilized to obtain the flame speed, v_o.

The model of the turbulent burning velocity[17] proposed by Damköhler, assumes that the rms fluctuation velocity, $u' = (\overline{u^2})^{1/2}$, dominates the flame front and can be correlated with the average flame speed, v_o. For fuel mixtures with relatively small laminar burning velocities and for high jet velocity, the burning velocity closely approaches the turbulent condition. This turbulent definition of the flame speed is assumed to apply in the mixing regime encompassed by the outer shear layer of the jet.

One selects a two-dimensional element of the outer shear region and makes the assumption that the region is of uniform thickness, equal to the mixing length, l. Within the layer, mass is transported primarily by fluctuating velocity components from the edge of the recirculation zone across the free shear layer to the outer ambient zone. Since reaction is rapid, the flame front is propagated at a velocity $V_n^* = v_o$ normal to the flow axis. In the plane of the shear element there are in addition two momentum flux components in the θ and z directions of magnitudes $U_z = \bar{U}_z + u_z$ and $U_\theta = \bar{U}_\theta + u_\theta$. Neglecting the contribution of turbulence generated by the flame, it is possible to treat this layer similar to the "surface layer" employed by Corrsin[18] in which the normal propagation velocity component is dependent only on the average magnitude of the total shear within the plane of the element.

As shown by Stewart[19]

$$v_o = \bar{V}_n^* = (\overline{u_\theta^2} + \overline{u_z^2})^{1/2} \tag{8}$$

Since $\bar{V}_n^*$ is also proportional to the mean turbulent propagation velocity in the radial direction, $(\overline{u_r^2})^{1/2}$, and since $v_o' \sim (\overline{u_r^2})^{1/2}$, the effective flame speed, is obtained

$$v_o = (\overline{u_\theta^2} + \overline{u_z^2})^{1/2} \tag{9}$$

The rms components $(\overline{u_\theta^2})^{1/2}$ and $(\overline{u_z^2})^{1/2}$ can be obtained in the following manner. From the definition of the mean stress tensor given in Eq. (5) the effective flame velocity in terms of the component diffusivities, is then

$$v_o = \left\{2\varepsilon_{rr}\left(\frac{\partial \bar{U}_r}{\partial r}\right) + \varepsilon_{rz}\left(\frac{\partial \bar{U}_r}{\partial z} + \frac{\partial \bar{U}_r}{\partial z}\right) + \varepsilon_{r\theta}\left(r\frac{\partial \bar{U}_{\theta/r}}{\partial r} + \frac{1}{r}\frac{\partial \bar{U}_r}{\partial \theta}\right)\right\}^{1/2} \tag{10}$$

Since this field is axisymmetric with respect to z, and the flow decays slowly with z, we may neglect gradients $\partial/\partial\theta$ and $\partial/\partial z$ with respect to $\partial/\partial r$ for the region under consideration. It follows for this analysis that $\partial\bar{U}_r/\partial r \ll \partial\bar{U}_\theta/\partial r$, $\partial\bar{U}_z/\partial r$. One obtains on simplification of Eq. (10)

$$v_o = \left\{\varepsilon_{rz}\left(\frac{\partial \bar{U}_z}{\partial r}\right) + \varepsilon_{r\theta}\left(r\frac{\partial \bar{U}_{\theta/r}}{\partial r}\right)\right\}^{1/2} \tag{11}$$

Introducing the diffusivities into the expression obtained earlier, there obtains for the average velocity of flame propagation

$$v_o = \left\{\left[l_{rz}^2\left(\frac{\partial \bar{U}_z}{\partial r}\right) + l_{r\theta}^2\left(r\frac{\partial \bar{U}/r}{\partial r}\right)\right] \times \left[\left(\frac{\partial \bar{U}_z}{\partial r}\right)^2 + \left(r\frac{\partial \bar{U}_{\theta/r}}{\partial r}\right)^2\right]^{1/2}\right\}^{1/2} \tag{12}$$

If it is assumed that $l_{r\theta}$ and l_{rz} are of the same order of magnitude, and the Prandtl modification for a free shear layer in which the over-all flow character in the mixing zone is determined by the dimensions of the zone and the greatest velocity across the zone, further simplifications of Eq. (12) are possible. Identifying the effective length across the zone as proportional to R, the radius of the jet, for a constant coefficient of mixing it follows

$$v_o = [K(\bar{U}_z + \beta\bar{U}_\theta)\cdot(\bar{U}_z^2 + \beta^2\bar{U}_\theta^2)^{1/2}]^{1/2} \tag{13}$$

where β is a correlation constant.

Simplifying and dividing by $\bar{U}_z$, the flame speed is obtained

$$v_o = [K(1+\beta S)\cdot(1+\beta^2S^2)^{1/2}\cdot\bar{U}_z^2]^{1/2} \tag{14}$$

where

$$S = \bar{U}_\theta/\bar{U}_z$$

Let $\Phi(S) = (1+\beta S)\cdot(1+\beta^2S^2)^{1/2}$, we find the speed

$$v_o = (K\Phi(S)\bar{U}_z^2)^{1/2} \tag{15}$$

where $\bar{U}_z$, the axial velocity of the jet is proportional to the mean velocity, $\bar{U}_\infty$, in the free shear layer and S is proportional to the maximum tangential velocity, V_T, at the exit of the jet.

We may infer from the behavior of the wake produced behind a bluff body, that the recirculation zone is modified by the extent of flame generated turbulence, the heat transfer released during combustion, the mean approach velocity, and the size of the obstacle. The composition and temperature within the zone are uniform due to large scale mixing. The influence of swirl on the recirculation distance can be inferred from studies on vortex breakdown obtained in water and air and related work of temperature profiles measurements of swirling jets. The length of the region is a function of the swirl ratio and some geometric length dimension which is dependent on the diameter of the recirculation bubble and, in turn, on the duct radius R.[9] From dimensional analysis, one obtains for the swirl recirculation length,

$$L/R = f(Re_\infty, S) \tag{16}$$

As the swirl goes to zero, this model must match the flow of a free nonswirling jet. For small swirl ratios, where reverse flow disappears, the mechanism of flame holding in the jet is appropriately controlled by a wall boundary-layer velocity gradient described by Lewis and Von Elbe and others,[20,21] thus $L/R \to f(Re_\infty)$ for $S = 0$. It was pointed out that critical swirl required for the development of a stagnation point in a vortex flow can be obtained approximately by considering a uniformly distributed cylindrical vortex source of finite length superimposed on an axial flow. An estimate of the induced axial velocity produced by this vortex sheet can be obtained by a method of Milne-Thompson.[28] For an infinite vortex tube superimposed on a moving flow, U_∞

$$U_{i_\infty} = \frac{1}{4\Pi}\int_o^\infty \frac{V_T \times \bar{r}\,ds}{r^3} \tag{17}$$

The solution to the integral is $U_{i_\infty} = -V_T/2$, the minus sign representing the direction of the induced velocity at the axis. Recirculation will occur at a critical swirl value of swirl

$$S_c = V_T/U_{i_\infty} > 2 \tag{18}$$

This represents the minimum tangential velocity that will induce a stagnation point, and represents a lower limit of the swirl value. Viscous effects will require that V_T/U_{i_∞} be somewhat smaller than 2. Bossel[4] obtained a solution to the laminar flow in a tube with rotation and suggests a critical swirl value

$$S_c = V_T/\bar{U}_\infty = (2)^{1/2} \tag{19}$$

Beer[2] suggests that the recirculation zone is proportional to the swirl value. Thus for the entire range of swirl, the blowoff velocity is obtained from Eq. (1)

$$U_{BO} = \frac{K'R(1+S^n)\Phi(S)\bar{U}_\infty^2}{\alpha} \tag{20}$$

The blowoff velocity is a function of the dimension of the holder and dependent on the approach Reynolds number for a bluff body. The flame holding capacity of the recirculation zone formed by a swirling jet can be characterized in a similar manner, with the jet diameter replacing the blockage dimension of the bluff body. The studies of Ranz[22] have indicated that with increasing swirl ratios, the recirculation zone may recede into the mouth of the jet and provide a smaller effective surface for flame holding. Thus it is maybe reasonable to expect that a region of stable operation occurs only when the recirculation zone has been formed. Under these conditions Eq. (20) applies. The limits of applicability of this model, may be anticipated when the

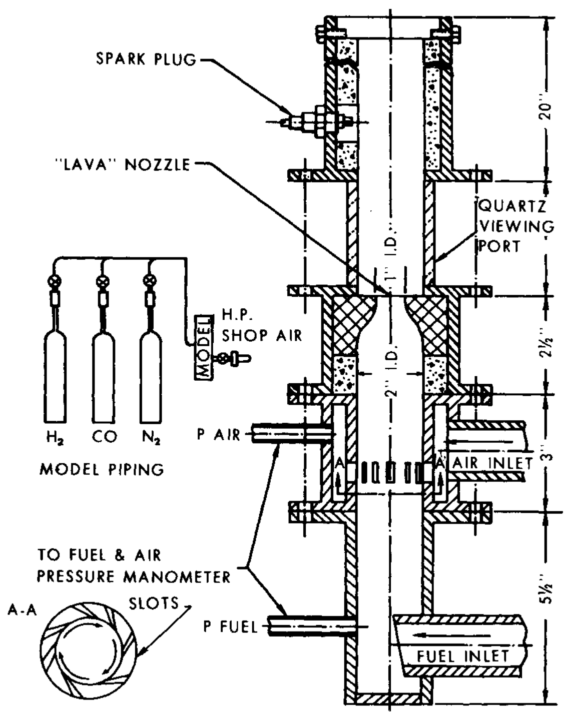

Fig. 2 Apparatus for swirl burner studies.

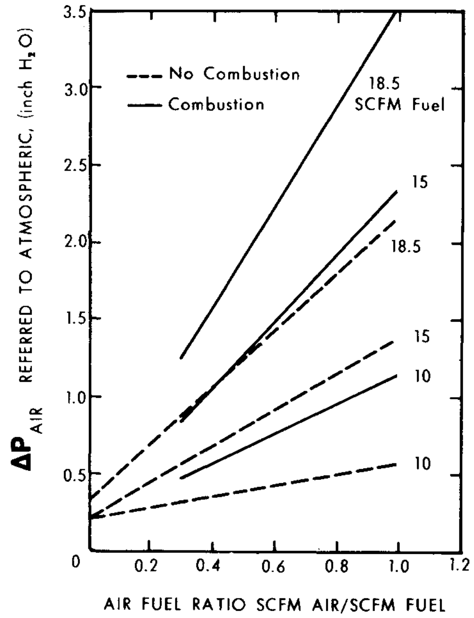

Fig. 3 Pressure loss measurements along axis of the burner. Note the effect of combustion on over-all pressure loss.

vortex bubble vanishes at small S values or due to transition of the flow to a Taylor column.

Experimental Procedure

In order to verify the results of the theory a preliminary experiment was performed in a small vortex burner shown schematically in Fig. 2. Fuel was supplied to the central duct and mixing occurred as the air was tangentially mixed with the air. Both fuel and air were metered through Fisher-Porter flowrators. The fuel composition was nearly similar to a low molecular weight Blast Furnace gas containing 71% N_2, 27% CO, and 1% H_2. The effective heat of combustion for the mixture was calculated to be 92 Btu/ft^3. Ignition was achieved at a high fuel flow rate, with a standard Autolite spark plug. The flame zone could be observed through a quartz viewing section above the nozzle section.

Blowoff experiments were performed with various fuel air mixtures. The limits of stable combustion were obtained by regulating the air flow at a fixed fuel rate to obtain either rich or lean blowoff limits. The data obtained from these experiments were used to determine the blowoff conditions for the burner.

Pressure loss measurements were also made across the burner chamber in the absence of and with combustion for selected fuel-air flow ratios. These results are shown in Figs 3 and 4 which show the major influence of swirl.

The minimum fuel flow rate required to support combustion was approximately 9.5 scfm. The flame length during stable combustion was approximately 32 in. in length with the lowest air fuel rates. With increasing swirl at the same fuel air ratio the length was reduced to approximately 2 in. At high swirl ratios the flame became quite noisy and lifted from the top of the burner nozzle. The approach Reynolds number was varied from 10^4 to 10^5.

Results

In Fig. 5, the combustion limits of the swirl burner in the present study are shown. The lean blowoff conditions exhibit a familiar steep behavior as already described[1,5] which indicates the swirl is a significant factor in the enhanced stability of the burning process. For premixed flames without swirl, blowoff occurred at 200 ft/in. for an air fuel ratio of 1.0.

Blowoff data were correlated with the maximum tangential velocity of the air at slit entrance and both lean and rich limits were replotted in Fig. 6, and show the effect of tangential velocity on the bowoff conditions. A least mean square fit to the data yields the blowoff velocity relationship

$$U_{BO} = C_1 V_T^{0.69} \tag{21}$$

This result is compared with the report of Potter, Berlad, and Wong[5] for propane and mixtures in a premixed burner with a rotating center shaft

$$U_{BO} = C_2 V_T^{0.73} \tag{22}$$

The data available from Alexander and Albright[1] did not

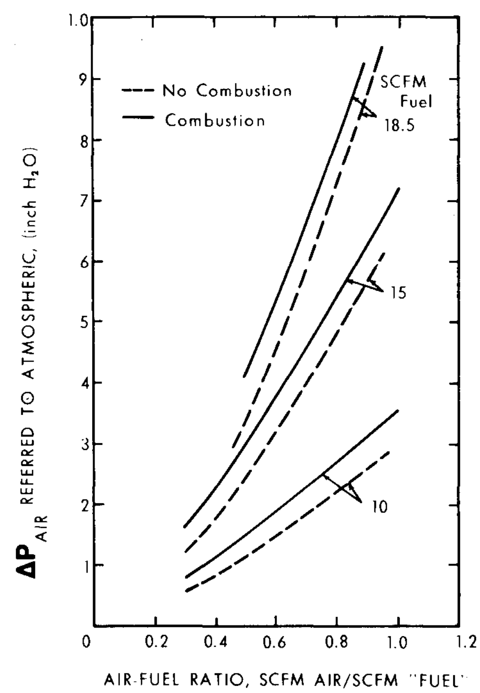

Fig. 4 Pressure loss across the swirl vanes as a function of swirl ratio.

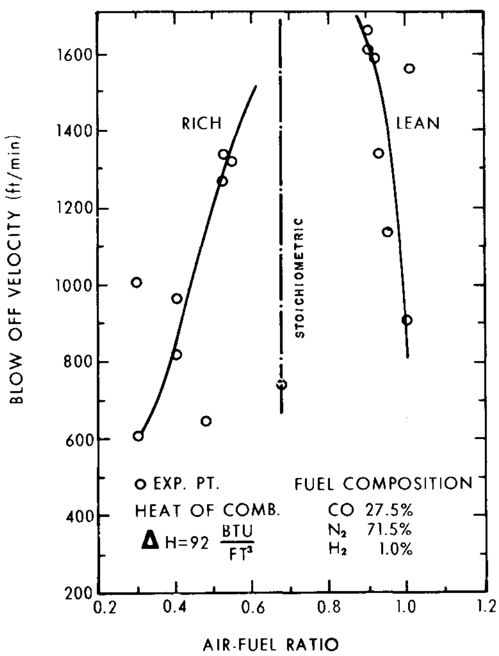

Fig. 5 Blowoff limits for the experimental burner showing both rich and lean limits.

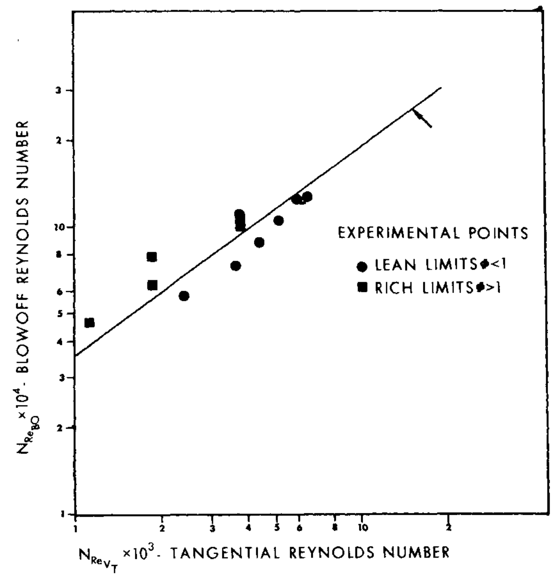

Fig. 6 Effect of swirl velocity on blowoff velocity as obtained from Fig. 5.

permit one to extract the effective tangential velocity obtained thus limiting our opportunity in this case for comparison. The results of Chigier and Chervinsky[24] have obtained experimental data for a 1-in.-diam swirling burner and have estimated the proportional increase in flame retention as a function of the swirl ratio. However, there is insufficient quantitative data to compare with the present experiment.

The data Fig. 5 were recalculated in terms of a mixing model developed in Eq. (21) for a $\beta = 1$, and for a recirculation distance independent of S. These results are presented in Fig. 7 and a least mean square fit of the data gives a slope of 0.56 when compared with the expected slope of unity. This disagreement of the eddy diffusivity should be re-examined in the light of additional experimental work. The fact that we have assumed β of unity $\Phi(S)$ may also be inaccurate. As yet there is insufficient detailed experimental data to adequately clarify this question. In a related study[25] it was indicated that the constants must be obtained by experiment. For a turbulent free shear flow behind a nonswirling axisymmetric wake correlation between u_θ' and u_z' is assumed to be in the neighborhood of 0.2–0.3.[26] For a superimposed tangential velocity component, the correlation is less certain. For large swirl ratio, the function (S) in Eq. (15) approaches $(\beta S)^2$ as a limited value. A plot of $S^2\bar{U}_\infty{}^2$ against the blowoff velocity, while obtaining a satisfactory coalescence of the data does not give the correct functional relationship as shown in Fig. 8. An empirical fit of the data obtains the relationship $(1 + S^2)$ which indicates that the function of swirl may be simpler than anticipated by the diffusivity model and dominated by the recirculation distance L. The results strongly suggest a constant eddy diffusivity is operative. Future work is required to elucidate this process.

The relative size of the jet to the diameter of the confining chamber suggests that local recirculation and attachment may occur inside the chamber as a result of the expansion. The effect can be estimated by means of the Newby-Thring or the Craya-Curtet criteria.[24] The fraction of recirculated flow due to expansion can be calculated from the Newby-Thring model indicates that for a nonswirling jet flow the mass entrained is of the order of 16%. A nonswirling jet attaches to the wall at a point 4.5 in. downstream of the nozzle. When swirl is introduced into the flow the expansion will be more pronounced and the entrainment may increase. However, it was observed in this work that at no time did the swirling flame attach to the wall. The flame zone was maintained within the duct and with an increase in swirl a noticeable decrease in the extent of the flame zone was observed. While these data represent only a preliminary study of the effect of a swirling jet in confined flow there are distinguishing features of this flow that suggest that recirculation problems are relatively insignificant with respect to the effect of swirl on the flame.

How well the results of blowoff compare with the model developed in this study is presumed to be a test of the adequacy of the model. It is not expected that the experiment will exactly satisfy the expression for blowoff for all conditions of swirl. What is indicated, however, is the ability to identify from the data a set of fluid dynamic variables, namely, the swirl ratio and the axial velocity issuing from the jet, that characterizes the process.

Discussion of Results

The limited data available on stability conditions in swirling flames obtained from these experiments is insufficient to completely substantiate the proposed fluid dynamic model for blowoff; and trends in the available data shown in Fig. 5 suggest that the fuel air ratio and physical properties of unburned

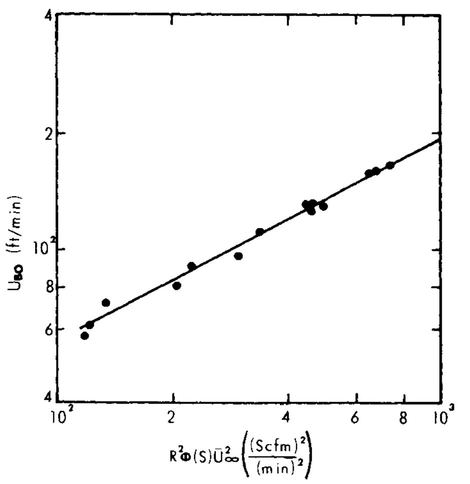

Fig. 7 Correlation of blowoff limits with Eq. (20) for $\beta = 1$.

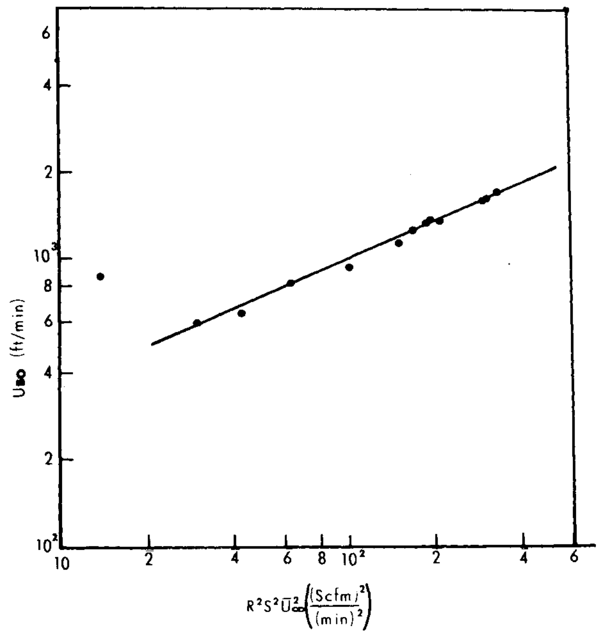

Fig. 8 Test of the limiting condition of high swirl number usin modified mixing length model.

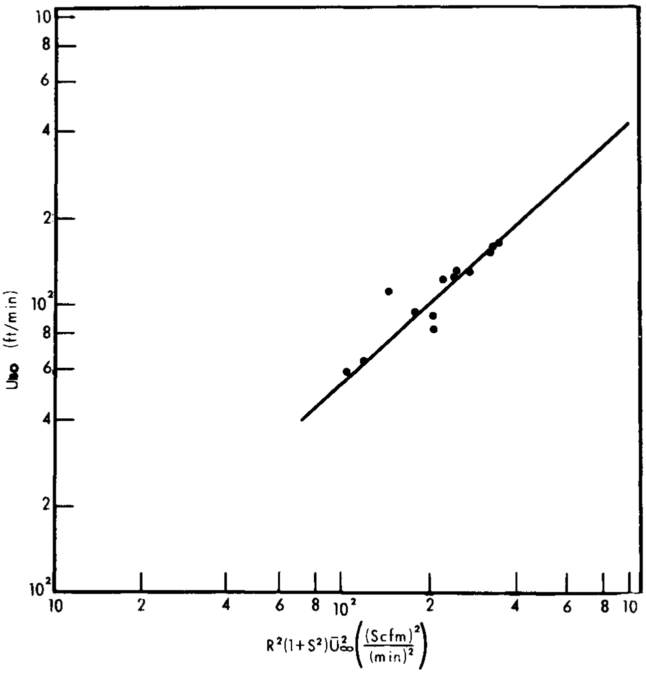

Fig. 9 Empirical correlation of swirl number with blowoff velocity for a constant eddy diffusivity in the shear region.

reactants may play a greater role than has been considered in this analysis in influencing the bowoff conditions of strongly swirling jets. However, from these results for an extended range of the fuel-air ratio for the given fuel mixture, the fluid dynamic variables do appear to exert a major influence on the flame holding properties of the jet. The highly turbulent nature of the outer zone of the expanding swirling jet can be likened to the wake regime behind a bluff body and indeed certain features between both systems are similar. As observed in the wake stabilized flame, the laminar flame speed is seen to be of secondary importance to the turbulence conditions that prevail in the mixing process in the combustion zone. Similar effects are observed for strong swirling flows without reaction. The tangential component of blow increases the degree of vorticity generated turbulence in the mixing zone above that observed in wake stabilized flames. While the effect of pressure has not been examined, insofar as it modifies the rate of reaction, turbulence per se appears to be the sole controlling factor in such flames. The exact role that chemistry plays has not been assessed in this study.

With respect to the diffusivity expression developed in Eq. (12) we note that Spaulding and Pun[27] have approximated the average diffusivity, ε_m, by an empirical relationship

$$\varepsilon_m = (KD^2/W)^{1/3}\rho^{2/3}[\dot{m}_\infty \bar{U}_\infty{}^2 + \dot{m}_T V_T{}^2]^{1/3} \tag{23}$$

where L is the area to length of the combustion chamber; V_T, U_∞ = velocity of the tangential and axial streams; $\dot{m}_T$, $\dot{m}_\infty$ = mass flow rates of tangential and axial streams; K = constant of mixing; ρ = average gas density; D = diameter of chamber; and W = length of chamber.

One can show that the diffusivity, ε_m, of Eq. (23) can be recast into the expression

$$\varepsilon_m = K(1+S)\bar{U}_\infty \cdot R \tag{24}$$

The swirl ratio, however, appears to the first power which is in disagreement with present theory. An empirical result was obtained for a free swirling annular jet which correlated with the limited available experimental data.[25] This is given by

$$\varepsilon_m = 0.00294R[(1+90S^2)]^{1/2}\bar{U}_\infty \tag{25}$$

This relation is compared with the derived diffusivity relationship employed in Eq. (14) where

$$\varepsilon_m = KR(1+\beta^2S^2)^{1/2}\bar{U}_\infty \tag{26}$$

suggesting evidence of comparable form. The agreement of this blowoff relationship with experimental data may indicate that the analysis has permitted the extraction of those variables important to design of swirl combustions.

Conclusions

A blowoff model for swirling flames based on an extended interpretation of the Prandtl mixing length is presented. The model proposes that the effective stability of such flames is dictated by the outer turbulent shear layer in which the tangential and axial fluctuating components are controlling the mixing process.

An analysis of the mixing process in the shear layer permitted the development of a general expression for the momentum diffusivity in the layer. The model assumed that the mixing lengths are comparable and the Prandtl mixing assumptions for the free shear layer are satisfied. There is general agreement between this model and available experimental data which suggests that the assumption of a directionally sensitive mixing length is not warranted. The diffusivity expression obtained on a phenomenological basis from a generalized treatment of the diffusivity function yields

$$\varepsilon_m = KR[1+\beta^2S^2]^{1/2} \cdot \bar{U}_\infty$$

This expression is compared with a current literature expression obtained by Rubel[25] and Spaulding.[27] The blowoff parameter developed to treat bluff body stabilization has been extended to swirling flow by assuming that the history of the swirling jet is similar to the wake process displayed behind a bluff body. The jet recirculation distance and the flame propagation velocity are modified to include the effect of swirl. For the present study the dynamic and geometric factors that influence swirl flame holding have been obtained and an expression for the blowoff velocity is developed

$$U_{BO} = K' \cdot R \cdot \Phi(S) \cdot \bar{U}_\infty{}^2/\alpha$$

where $\phi(S)$ is dependent upon the degree of swirl insofar as it affects the extent of the recirculation zone and mixing length. Agreement with experimental data appears to verify the form of the expression.

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Ignition and Global Combustion Models for Clouds of Boron Particles

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It is shown that the single particle ignition model proposed by Merrill King for the ignition of boron particles can be modified and extended to predict with reasonable accuracy the ignition delay times of single boron particles exposed to an oxidizing atmosphere. Provisions were also included to account for the effect of changes in system pressure, particle diameter, surrounding gas temperature, and the mole fraction of water vapor in the gas. The single particle ignition model is extended to clouds of particles by attempting to account for such factors as interparticle radiation and the finite amount of gas surrounding each particle. This model predicted ignition of particle clouds under certain conditions which would not favor the ignition of single particles. Finally, a global combustion rate is derived for a cloud of boron particles burning under steady-state conditions. This combustion rate was obtained from an analysis of the data obtained from a series of experimental runs performed in a one-dimensional flow reactor.

Nomenclature

A = surface area, cm^2
c_p = specific heat, cal/g-°K
d = particle diameter, cm
E = activation energy for evaporation
ΔH_f = heat of fusion of boron, cal/g
ΔH_{vap} = heat of vaporization of boron oxide, cal/mole
h = convective heat-transfer coefficient, cal/cm^2-sec-°K
k = thermal conductivity, cal/cm-sec-°K
M = molecular weight
m = mass, g
P = pressure, atm
Pr = Prandtl number of the gas
Q = heat of combustion of boron with diatomic oxygen to form liquid B_2O_3, cal/mole; total energy, cal
R = molar rate of consumption of boron, g-mole/sec; gas constant
R_E = molar rate of evaporation of boron oxide, gm-mole/sec
Re = Reynolds number based on particle diameter and relative velocity between the gas and particle
r = radius, cm
Δr = oxide film thickness, $r_2 - r_1$, cm
T = temperature, °K
t = time, sec
w = particle cloud mass density, g/cm^3
X_{H_2O} = mole fraction of H_2O present in the gas
α = surrounding absorptivity ($=1.0$)
ε = particle emissivity ($=0.8$)
λ = particle burning rate, cm/sec
ρ = density, g/cm^3
σ = Stefan-Boltzmann constant

Subscripts and Superscripts

b = boil, burned
f = final
g = gas
i = initial
m = melt
p = particle
RAD = radiation environment
1 = particle; first visible zone
2 = particle + oxide layer; steady-state combustion zone

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